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ADVANTAGES OF A CRYOGENIC ENVIRONMENT FOR EXPERIMENTAL INVESTIGATIONS OF CONVECTIVE HEAT TRANSFER

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NOMENCLATURE

c_p, specific heat [J kg^{-1} K^{-1}];
g, gravitational constant [m s^{-2}];
L, characteristic length [m];
p, absolute pressure [Pa];
P, fan power [W];
T, absolute temperature [K];
ΔT, $(T_w - T_\infty)$ [K];
V, velocity [m s^{-1}].

Greek symbols
β, expansion coefficient [K^{-1}];
ρ, density [kg m^{-3}];
μ, dynamic viscosity [kg s^{-1} m^{-1}];
ν, kinematic viscosity, μ/ρ, [m^2 s^{-1}].

Dimensionless quantities
Gr, Grashof number, $\rho^2 g\beta\Delta T L^3/\mu^2$;
Nu, Nusselt number, hL/k;
Pr, Prandtl number, $\mu c_p/k$;
Re, Reynolds number, $\rho VL/\mu$;
Ri, Richardson number, Gr/Re^2.

Subscripts
w, wall condition;
∞, free stream condition;
f, referenced to film temperature.

1. INTRODUCTION

EXPERIMENTALISTS engaged in the investigation of convective heat transfer phenomena often have difficulty in obtaining sufficiently large Reynolds numbers, *Re*, and/or Grashof numbers, *Gr*. Large scale tests or prototype studies are often too expensive and are difficult to resolve accurately. On the other hand, sufficiently large Reynolds and/or Grashof numbers are difficult to generate in controlled laboratory investigations which use small geometrically similar models. In both cases, convective heat transfer rates are often hard to separate from the radiative heat transfer.

An example of one such problem is the determination of the convective loss from the receiver of the 10 MW$_e$ solar-thermal electric plant in Barstow, California. The receiver is a right-circular cylinder with a diameter D of approximately 7 m and a height L of approximately 13 m. The solar flux is absorbed on the external cylindrical surface, and the convective energy loss from this surface is of interest. Even with a wind of only several meters per second, the Reynolds numbers for the crossflow over the receiver are of the order of 10^6. Characteristic Grashof numbers are also large, on the order of 10^{13}. The Richardson number, $Ri = Gr/Re^2$ is on the order of one; hence, it appears to be a combined convection phenomenon. The ratios of absolute surface temperature, T_w, to ambient temperatures, T_∞, are of the order of three; thus, property variations across the boundary layer are of importance. The experimental tasks are to simultaneously generate both large Reynolds and Grashof numbers, and to generate large ratios of T_w/T_∞ without the results being masked by radiative heat transfer.

Cryogenic temperatures have been used in order to generate high Reynolds numbers for research in aerodynamics. On the other hand, the advantages of low temperature environment in convective heat transfer research have not been realized. An analysis of modeling options with cost implications is presented in this paper which clearly shows the advantages of using a cryogenic environment. Emphasis is placed on heat transfer in gases at low Mach numbers.

2. MODELING CHARACTERISTICS AND OPTIONS

Consider the conditions which must be satisfied for geometric, dynamic and thermal similarity between model and prototype. The governing dimensionless groups can be deduced from a dimensional analysis of the general, compressible forms of the continuity, momentum and energy equations, the equation of state, and the property relationships. The simplifying assumptions are: a laminar flow of a Newtonian fluid, a perfect gas, negligible gas absorption, negligible viscous dissipation, negligible work done by compression, a negligible influence of Mach number, and the dependent variables c_p^*, μ^*, and k^* are general functions of only the dimensionless temperature: $c_p^* = f_1(T^*)$, $k^* = f_2(T^*)$ and $\mu^* = f_3(T^*)$ where the asterisk denotes a dimensionless ratio. If one assumes geometrically similar models are used, an isothermal body at T_w, an isothermal ambient at T_∞, a uniform free stream velocity, V, and a fixed orientation between the velocity, V, the acceleration of gravity, g, and the geometry of interest, a dimensional analysis of the governing equations shows that the local Nusselt number in combined convection is dependent on

$$Nu = f(\mathbf{S}^*, Re, Gr, Pr, T_w/T_\infty) \quad (1)$$

where $\mathbf{S}^*$ is the vector denoting the dimensionless surface location of interest. Experience has shown that equation (1) is valid for both laminar and turbulent flows. Including the additional group, T_w/T_∞, enables one to arrive at equation (1) without making the Boussinesq approximation. The form of equation (1) is not unique. Alternative forms include the use of the Richardson number, Gr/Re^2, the Rayleigh number, $GrPr$, or the Froude number, $Fr = V^2/gL$, in place of the Grashof number.

Gases are of interest, and gaseous nitrogen is used in the cryogenic test facility which is described later. The Prandtl number is approximately 0.7 for many gases and is essentially a constant—independent of both temperature and pressure. Hence, the influence of the Prandtl number need not be resolved. The results are, of course, applicable only to fluids with Prandtl numbers near 0.7 with this modeling strategy. At homologous points on geometrically similar bodies, the local heat transfer rate or coefficient is dependent on T_w/T_∞, Re, and Gr. The advantages of a cryogenic, variable temperature facility in deducing the influences of these dimensionless variables will be considered.

Consider first the reduction in radiant heat transfer which is effected with a cryogenic facility. Most heat transfer models have metallic surfaces. In the temperature range of interest ($T < 1300$ K), the total emissivity of pure metals is directly proportional to temperature; hence, a T_∞ of 80 K with equal values of T_w/T_∞ results in a reduction in emitted energy by a factor of approximately 1000†.

Consider next the influence of variable properties. A common practice in many experimental and analytical studies is to neglect all property variations other than the essential density difference which must be considered in buoyancy driven flows. Many applications exist, however, where T_w/T_∞ is appreciably different than unity. Experimentalists also use values of T_w/T_∞ which are noticeably different from unity, and this appears to be a significant source of discrepancies in the resulting data. Consider the advantage of a cryogenic facility in an investigation of the influence of variable properties—for example, an application such as a high temperature solar receiver which requires $T_w/T_\infty = 4$ for perfect modeling. The resulting model temperatures with T_∞ equal to 80 and 320 K are 320 and 1280 K, respectively! Even at 1000 K insulators are less effective, and good insulators have higher thermal mass, many common materials are no longer usable, suitable instrumentation is more expensive and less accurate, and radiative heat transfer is excessive.

2.1. Re—*Forced convection*

The basic experimental problem has two main aspects: obtaining large Reynolds numbers and minimizing costs. To further simplify the analysis, assume that the viscosity μ varies with temperature as T^m‡ and the fan power P is proportional to $\rho V^3 L^2$. An examination of the Reynolds number shows

$$Re \propto \frac{p}{T^{1.9}}(VL). \qquad (2)$$

Thus, if cost is unimportant, elevated pressures, reduced temperatures, high velocities, and large lengths can be used individually or in combination in order to achieve large Reynolds numbers.

The overall cost of a test facility, as well as the operating costs, is strongly coupled to the fan-power. Thus, the parameter P/Re is an important indicator of the cost of obtaining a given Reynolds number.

$$P/Re \propto \mu V^2 L \propto T^{0.9}(V^2 L). \qquad (3)$$

Equation (3) shows that low temperatures, small characteristic lengths and, above all, low velocities are desirable to minimize this cost. P/Re is seen to be independent of pressure. Decreasing the temperature is the only means of increasing the Reynolds number and simultaneously decreasing the relative drive cost. On the other hand, the least cost effective way of obtaining large Reynolds numbers is to use high velocities.

Figure 1 shows the influence of temperature on the Reynolds number and the Reynolds number cost P/Re with constant velocity and pressure. The Reynolds number is not only increased by a factor of 14 at 80 K, but the cost of a given Reynolds number is decreased by a factor of 3.5. A variable temperature facility has the additional advantage, in the deduction of empirical correlations, that the Reynolds number can be varied greatly without changing either L or V.

2.2. Gr—*Natural convection*

The influences of buoyancy are dependent on the Grashof number, $\rho^2 g\beta\Delta T L^3/\mu^2$. The fundamental problem in modeling this influence in large systems is accounting for the change in L^3. For example, a one-tenth size model results in a factor of 1000 which must be balanced by appropriate changes in $\rho^2 g\beta\Delta T/\mu^2$. Since the volume coefficient of expansion β is $1/T$ for a perfect gas,

$$Gr \propto \frac{p^2}{T^{4.8}}(\Delta T)L^3 \qquad (4)$$

where ΔT is the difference between the surface temperature and the ambient temperature, $(T_w - T_\infty)$. Equation (4) shows that the Grashof number increases with the square of the absolute pressure; hence, significant gains can be effected by the use of elevated pressure. However, the use of elevated pressures in natural convection research is particularly expensive because large characteristic lengths are requisite.

The strong influence of the ambient temperature on the Grashof number, which equation (4) clearly reveals, is shown in Fig. 2. Constant values of p, ΔT, and L are assumed. The Reynolds number is provided for comparison. The use of cryogenic temperatures is a good technique to obtain high Reynolds numbers but an even better way of obtaining large Grashof numbers. The Grashof number ratio at 80 K is increased by a factor of 800. If the property $\rho^2\beta/\mu^2$ is based on

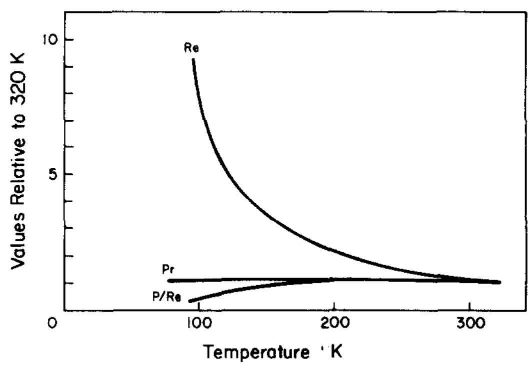

FIG. 1. The influence of temperature on Re, Pr and P/Re.

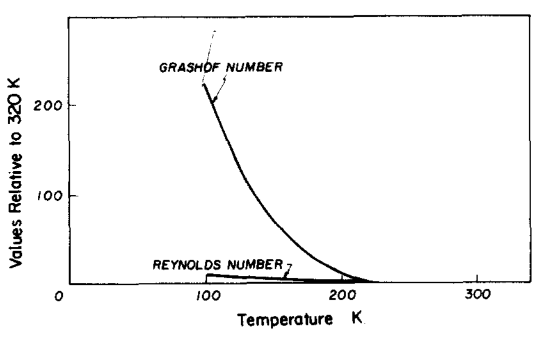

FIG. 2. The influence of temperature on Re and Gr.

† All quantities in the analysis which follows are references to 320 K — a typical operating temperature of a recirculating tunnel.

‡ m is approximately 1.0 at 100 K, 0.8 at 300 K, and 0.6 at 1000 K. A value of 0.9 is used in the discussions.

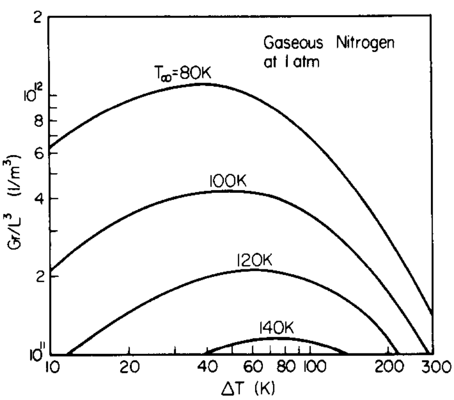

FIG. 3. The influence of ΔT and T_∞ on Gr/L^3.

the film temperature,† the influence of ambient temperature becomes strongly coupled to ΔT if ΔT is large. Specifically, equation (4) becomes

$$Gr \propto \frac{p^2 \Delta T L^3}{\left(T_\infty + \dfrac{\Delta T}{2}\right)^{4.8}}. \tag{5}$$

The influence of ΔT on this Grashof number is given in Fig. 3 for several values of T_∞. These results clearly show that an optimum temperature difference exists which maximizes the Grashof number. If equation (5) is differentiated with respect to ΔT and the derivative is set equal to zero, one obtains

$$\Delta T_{opt} = T_\infty/1.9. \tag{6}$$

The maximum obtainable Grashof number follows as

$$Gr_{max} = gL^3/(2.4\nu^2). \tag{7}$$

Gr_{max} based on published property data lies within 4% of equation (7) over the temperature range $80 < T_\infty < 320$ K. The ratio of the maximum Grashof number for gaseous nitrogen at 80 K relative to the maximum Grashof number for gaseous nitrogen or dry air at 320 K is approximately 200.

Figure 3 shows the futility of trying to cover a range of the parameter Gr by changing ΔT. On the other hand, it is possible to cover a range in Gr of more than two orders of magnitude by varying T_∞ between 80 and 300 K—a significant advantage of a variable temperature, cryogenic facility.

2.3. Ri—*Combined convection*

If the influences of both natural and forced convection are of importance, Gr/Re^2, the ratio of the buoyant force to the inertia force, is of the order of 1. Both Gr and Re must be considered in this regime; thus, the basic problem is compounded. Obtaining a large value of Re^2 is of no avail if corresponding magnitudes of Gr cannot be generated. The definition of the Richardson number referenced to the film temperature gives

$$Ri = \frac{Lg\Delta T}{V^2 T_f} = \frac{Lg}{V^2}\,\frac{2(T_w/T_\infty - 1)}{(T_w/T + 1)}. \tag{8}$$

Since the velocity V can be reduced if the inertia force is too large, the problem is to obtain sufficiently large values of $\Delta TL/T_f$ to fully realize the Reynolds number capability of the facility. Equation (8) shows that Ri is independent of pressure; hence, a variable pressure facility would be of no benefit. The potential gains with a variable temperature cryogenic facility are again evident. For example, varying ΔT between 0 and 80 K, covers a range of Ri which is three times greater with $T_\infty = 80$ K than that covered with $T_\infty = 320$ K.

† The film temperature is a more universally employed datum, and it greatly reduces the influence of T_w/T_∞ in laminar flow regimes [1].

3. CRYOGENIC FACILITY

The ideal facility for studying forced, free and combined convection at high Reynolds and/or Grashof numbers would be an elevated pressure, cryogenic facility. Since large cost savings can be realized by utilizing an ambient pressure facility, a pure cryogenic facility was constructed at the University of Illinois at Urbana-Champaign (UIUC). This wind tunnel is a recirculating, liquid-nitrogen cooled design which uses gaseous nitrogen as the working fluid. The tunnel has a rectangular test section which is 1.2 m in height by 0.6 m. The large vertical dimension was chosen to facilitate the generation of large Grashof numbers. A 11.5 kW, variable speed DC motor drives the two 0.5 m dia. cast aluminum fans while four sets of 90° turning vanes direct the flow around the circuit. Fan swirl is reduced by a honeycomb flow straightener and a settling screen which reduces the turbulent intensity and creates a more even velocity distribution. A 0.4 m thick wall of closed-cell polyurethane foam is sandwiched around a vapor barrier to reduce heat gain and air infiltration. The facility is capable of operating at any temperature between 80 and 350 K and at velocities between 0 and 8 m s^{-1}. The maximum obtainable Grashof number based on a 1 m length and an optimum ΔT is 10^{12}, and the maximum Reynolds number based on a 0.2 m length is 1.5×10^6. Design details and flow characteristics are given in ref. [2].

Appreciable data have already been obtained from the UIUC facility. For example, a recently completed natural convection study [1] showed that: (i) a range of approximately three orders of magnitude in the Rayleigh number could be covered with a single model by changing T_∞, (ii) excellent agreement is obtained with published correlations in both the laminar and turbulent regimes if T_w/T_∞ is approximately unity, and (iii) the parameter T_w/T_∞ strongly influences the Nusselt number. These data cover the range $1 < T_w/T_\infty < 2.6$. A combined convection, variable property study is currently in progress which is designed to determine the influences of Re, Ri, and T_w/T_∞ for heat transfer from a vertical right-circular cylinder—a model of the 10 MW$_e$ Barstow solar receiver.

4. CONCLUSIONS

A cryogenic environment provides a means of obtaining, *simultaneously*, large increases in both the Reynolds number and the Grashof number; hence, it provides an excellent tool for forced, natural and combined convective heat transfer research. The Reynolds and Grashof numbers are increased with an ambient temperature of 80 K by factors of approximately 14 and 200, respectively, over those obtainable in a room temperature facility. The cryogenic environment virtually eliminates the influences of radiative heat transfer.

The ability to vary the temperature in the test section greatly increases the range in the Reynolds and Grashof numbers that can be investigated with fixed model and test section dimensions. The cryogenic facility also provides an excellent environment for the investigation of the influences of property variations across the boundary layers.

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